Theorem 2. For the asymptotic stability of the trivial solution of system (3.1) (and, consequently, of the original system (0.1) with $n = 1$ and fourth-order resonance), it is necessary and sufficient to satisfy the inequalities

$$a_1 < -\sqrt{a_2^2 + b_2^2 - b_1^2}$$
$$(a_1 \neq -\sqrt{a_2^2 + b_2^2 - b_1^2}, \quad b_1^2 < a_2^2 + b_2^2)$$

Note. The instability conditions given by the theorem extend, in particular, to Hamiltonian systems. In fact, as is easily verified, from the canonicity conditions for system (3.1) it follows that $a_1 = 0$, therefore, on the basis of the theorem proved, we conclude that the periodic solution of the original canonic system is unstable when $b_1^2 < a_2^2 + b_2^2$.

The author thanks V. V. Rumiantsev and the participants of a seminar conducted by him for discussions on the paper.

REFERENCES

1. Gol'tser, Ia. M. and Kunitsyn, A. L., On stability of autonomous systems with internal resonance. PMM Vol. 39, № 6, 1975.
2. Liapunov, A. M., General Problem of the Stability of Motion. Collected Works, Vol. 2, Moscow, Izd. Akad. Nauk SSSR, 1956.
3. Kamenkov, G. V., Selected Works, Vol. 1. Stability of Motion. Oscillations. Aerodynamics. Moscow, "Nauka", 1971.
4. Malkin, I. G., Theory of Stability of Motion. Moscow, "Nauka", 1966.
5. Briuno, A. D., Analytic form of differential equations. Tr. Mosk. Matem. Obshch., Vol. 25, 1971.
6. Kunitsyn, A. L., On the stability of periodic motions under resonance. PMM Vol. 39, № 1, 1975.

Translated by N. H. C.

UDC 62-50

STABILIZATION IN A LINEAR DIFFERENTIAL GAME WITH INTEGRAL CONSTRAINTS ON THE PLAYERS' CONTROL RESOURCES

PMM Vol. 40, № 3, 1976, pp. 439-445

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(Received September 17, 1975)

We examine an encounter game problem for a linear controlled system. We assume that the controls of the first and second players are subject to integral constraints. Using the idea of control with a guide [1] and the methods of stabilization theory [2], we construct a stabilized control procedure ensuring a stable encounter of the motions generated by it with a specified target set. The contents of the paper is related to the investigations in [1, 3 — 5].

1. Let the motion of a controlled system be described by the vector differential equation

$$dx \,/\, dt = Ax + Bu + Cv, \quad x[t_0] = x_0 \tag{1.1}$$

Here x is the system's n-dimensional phase vector; u and v are the players' controls of dimension m; A, B and C are constant matrices of appropriate dimensions. The realizations $u[t]$ and $v[t]$ of the controls are constrained by the relations

$$I_u(t_0, \infty) \leqslant \mu[t_0], \quad I_v(t_0, \infty) \leqslant \nu[t_0] \tag{1.2}$$

$$I_u(a, b) = \left(\int_a^b \|u[\xi]\|^2 d\xi\right)^{1/2}, \quad I_v(a, b) = \left(\int_a^b \|v[\xi]\|^2 d\xi\right)^{1/2}$$

on $[t_0, \infty]$. Here $\mu[t_0]$ and $\nu[t_0]$ are constraints on the resources of the controls u and v. We assume that the variations of the quantities $\mu[t]$ and $\nu[t]$ are determined by the expendable resources (1.3)

$$\mu^2[t+\Delta] = \mu^2[t] - I_u^2(t, t+\Delta), \quad \nu^2[t+\Delta] = \nu^2[t] - I_v^2(t, t+\Delta)$$

The set

$$M = \{z = (t, \mu, \nu, x) : (t, x) \in M^*, \ \mu \geqslant 0, \ \nu \geqslant 0\}$$

is specified in the Euclidean space R^{n+3} of points $z = (t, \mu, \nu, x)$ Here M^* is a closed set of the space of vectors (t, x), contained in the domain $D = \{(t, x) : t_0 \leqslant t \leqslant \vartheta, \ x \in R^n\}$ (ϑ is the instant limiting the game's duration). The problem facing the first player is the construction of a position action method U^* which would ensure him that the point $(t, x_\Delta[t])$ hits M_ε^* for any approximate motion $x_\Delta[t]$ generated by this action method; here M_ε^* is the ε-neighborhood of set M^*.

The encounter problem for a differential game with integral constraints was solved in [5] by using the general scheme of control with a guide [1]. This solution possessed the property of stability relative to small measurement errors in the phase vector. However, when the game's duration is sufficiently long the stability of the solution was ensured only under extremely small bounds on the step size of the approximation scheme and on the phase vector measurement error, because the estimate of the mismatch between the motion $x_\Delta[t]$ of the actual system and the motion $w_\Delta[t]$ of the guide was exponential in nature. In the present paper we construct the required control method U^* in the form of a stabilized procedure of control with a guide in order to improve the estimate of the mismatch. Such a solution of game problems with geometric constraints on the players' controls was proposed in [3, 4].

We shall assume that the following conditions are satisfied.

Condition A. A u-stable bridge W_u^ϑ exists. (The definition of a u-stable bridge W_u^ϑ is taken here in the sense of [5]).

Condition B. The system described by the equation

$$ds / dt = As + Br, \quad s \in R^n, \quad r \in R^m \tag{1.4}$$

is stabilizable [2]. Here r is a control vector.

The fulfillment of Condition B implies the existence of a linear vector function $r_0(s) = R_0 s$ such that the zero solution of system (1.4) is asymptotically Liapunov stable. Then, by Liapunov's theorem (see [2]), for any preassigned negative-definite quadratic form $\beta(s)$ we can find a positive-definite quadratic form $\lambda(s)$ whose total derivative, by virtue of the equation

$$ds / dt = As + BR_0 s$$

satisfies the equality

$$d\lambda \,/\, dt = (\partial\lambda/\partial s)' \ (As + BR_0 s) = \beta\ (s) \tag{1.5}$$

Here and below the prime denotes transposition.

2. Let us determine the approximation procedure of control with a guide for the first player, solving the encounter problem. This procedure assumes the presence of an auxiliary construction consisting of the u-stable bridge W_u^{ϑ}, a rule for choosing a guide on this bridge and a rule of stabilized aiming on the guide's motion. Following the scheme of control with a guide [1], together with the actual system described by relations (1.1) and (1.3) we consider an auxiliary ideal system whose motion $w\,[t]$ is described by the equation

$$dw \,/\, dt = Aw + Bu_* + Cv_*, \quad w\,[t_0] = w_0 \tag{2.1}$$

Here the realizations $u_*\ [t]$ and $v_*\ [t]$ of the controls are constrained on $[t_0, \infty)$ by the relations

$$I_{u_*}\ (t_0, \infty) \leqslant \mu_*\ [t_0], \quad I_{v_*}\ (t_0, \infty) \leqslant \nu_*\ [t_0]$$
$$\nu_*\ [t_0] = \nu\ [t_0], \quad \mu_*^2\ [t_0] = \mu^2\ [t_0] - \eta^2\ [t_0]$$

where $\eta\ [t_0] > 0$ is the resource used in the control procedure for stabilizing the motion $s\ [t] = x\ [t] - w\ [t]$ on $[t_0, \vartheta]$.

We introduce auxiliary concepts and notation. Let $z_* = (t_*, \mu_*, \nu_*, w)$ be some game position and $v_*\ [t]$ be a realization of the second player's control, admissible for this position, i.e. $v_*\ [t]$ is a summable function satisfying the inequality $I_{v_*}\ (t_*, \infty) \leqslant \nu_*$. By the symbol $G^{(u)}\ (z_*, v_*\ [\cdot])$ we denote the closure of the set of points $z = (t, \mu\ [t], \nu\ [t], w\ [t])$ of the form

$$t_* \leqslant t, \quad 0 \leqslant \mu^2\ [t] \leqslant \mu_*^2 - I_{u*}^2\ (t_*, t)$$
$$0 \leqslant \nu^2\ [t] \leqslant \nu_*^2 - I_{v*}^2\ (t_*, t)$$
$$w\,[t] = w + \int_{t_*}^{t} (Aw\,[\sigma] + Bu_*\,[\sigma] + Cv_*\,[\sigma])\, d\sigma$$

where $u_*\ [\sigma]\ (\sigma \geqslant t_*)$ are all possible summable functions satisfying the inequality $I_{u_*}\ (t_*, \infty) \leqslant \mu_*$.

We introduce into consideration the function

$$u\ [s;\ t,\ \delta;\ u_*\ [\cdot]] = p\ [s;\ t,\ \delta;\ u_*\ [\cdot]] + r\ (s) \tag{2.2}$$
$$v_*\ [s;\ t,\ \delta;\ v\ [\cdot]] = q\,[s;\ t,\ \delta;\ v\ [\cdot]]$$

where the quantities in the right-hand sides are determined from the relations

$$p\ [s;\ t,\ \delta;\ u_*\ [\cdot]] = 0, \quad q\ [s;\ t,\ \delta;\ v\ [\cdot]] = 0, \quad t = t_0$$
$$(\partial\lambda\ (s) \,/\, \partial s)'\ Bp\ [s;\ t,\ \delta;\ u_*\ [\cdot]] = \min_p\ (d\lambda\ (s) \,/\, ds)'\ Bp$$
$$\|\, p \,\| = \delta^{-1/2}\, I_{u_*}\ (t - \delta,\ t), \quad t \in [t_0,\ \vartheta], \quad 0 < \delta \leqslant t - t_0$$
$$(\partial\lambda\ (s) \,/\, \partial s)'\ Cq\ [s;\ t,\ \delta;\ v\ [\cdot]] = \max_q\ (\partial\lambda\ (s) \,/\, \partial s)'\ Cq$$
$$\|\, q \,\| = \delta^{-1/2} I_v\ (t - \delta,\ t), \quad t \in (t_0,\ \vartheta], \quad 0 < \delta \leqslant t - t_0$$
$$r\ [s] = R_0 s$$

(t_0 is the initial instant). Here $u_*\ [\cdot]$ and $v\ [\cdot]$ are square-summable functions given on $[t - \delta,\ t]$ and $s \in R^n$. Let the initial position $z\ [t_0] = (t_0, \mu\ [t_0], \nu\ [t_0],\ x\ [t_0])$.

and the u-stable bridge W_u^ϑ be specified. Assuming that bridge W_u^ϑ is closed in the space R^{n+3}, we select the point $z_*[t_0] = (t_0,\ \mu_*[t_0],\ \nu_*[t_0],\ w[t_0]) \in W_u^\vartheta$ closest to the point $z[t_0]$. This point $z_*[t_0]$ is the guide's position at the initial instant $t = t_0$.

Let $\Gamma_\Delta = \{t_0,\ t_1,\ \ldots,\ t_N = \vartheta\}$ be a partitioning of the interval $[t_0,\ \vartheta]$ by instants t_i into half-intervals of equal length Δ. When determining the first player's guide-control procedure we assume that system (2.1) is at the disposal of the first player. This player, having available both the choice of control $u[t]$ in the system (1.1), (1.3) as well as the choice of controls $u_*[t]$ and $v_*[t]$ in system (2.1), has the opportunity of computing exactly the realized value of the phase vector $w_\Delta[t], t \in [t_0, \vartheta]$ of system (2.1). However, he computes the value of the phase vector $x_\Delta[t]$ and, together with it, of the vector $s_\Delta[t] = x_\Delta[t] - w_\Delta[t]$ with an error $\Delta s_\Delta[t]$ and he constructs the controls $u[t]$ and $v_*[t]$ on the half-interval $[t_i,\ t_{i+1})$ on the basis of the vector

$$s_\Delta^*[t_i] = s_\Delta[t_i] + \Delta s_\Delta[t_i] \quad (\|\Delta s_\Delta[t_i]\| \leqslant \xi)$$

Let us describe the guide-control procedure in detail. On the first half-interval $[t_0, t_1)$ the motion $\varkappa(t) \equiv (\mu_\Delta[t],\ \nu_\Delta[t], x_\Delta[t])$ of the system (1.1), (1.3) with the initial condition $\varkappa(t_0) = (\mu[t_0],\ \nu[t_0],\ x[t_0])$ is generated by a constant control of the first player

$$u[t] = u[s_\Delta^*[t_0];\ t_0,\ \Delta;\ u_*[\cdot]] \quad (t_0 \leqslant t < t_1)$$

in pair with a certain admissible realization $v[t]$ $(t \geqslant t_0)$ of the second player's control. Here

$$s_\Delta^*[t_0] = s_\Delta[t_0] + \Delta s_\Delta[t_0]$$
$$s_\Delta[t_0] = x_\Delta[t_0] - w_\Delta[t_0], \quad w_\Delta[t_0] = w[t_0]$$

Let

$$v_*[t] = v_*[s_\Delta^*[t_0];\ t_0,\ \Delta;\ v[\cdot]] \quad (t_0 \leqslant t < t_1)$$

We choose the guide's position $z_{*\Delta}[t_1]$ at the instant t_1 from the condition

$$z_{*\Delta}[t_1] \in W_u^\vartheta(t_1) \cap G^{(u)}(z_{*\Delta}[t_0],\ v_*[\cdot])$$

assuming that this intersection is not empty. Here

$$z_{*\Delta}[t_0] = z_*[t_0]$$
$$W_u^\vartheta(t) = \{z : z = (t,\ \mu,\ \nu,\ x),\ z \in W_u^\vartheta\}$$

Further, on the half-interval $[t_1,\ t_2)$ the motion $\varkappa(t)$ of system (1.1), (1.3) with the initial value $\varkappa(t_1)$ is generated by the constant control

$$u[t] = u[s_\Delta^*[t_1];\ t_1,\ \Delta;\ u_*[\cdot]] \quad (t_1 \leqslant t < t_2)$$

where $u_*[\sigma]$ $(t_0 \leqslant \sigma < t_1)$ is the first player's control which in pair with $v_*[\sigma]$ $(t_0 \leqslant \sigma < t_1)$ generates the guide's position $z_{*\Delta}[t_1]$.

The game position $z_\Delta[t_2]$ is realized as a result of choosing the control $u[t]$ $(t_1 \leqslant t < t_2)$ and some control $v[t]$ $(t \geqslant t_1)$. We choose

$$v_*[t] = v_*[s_\Delta^*[t_1];\ t_1,\ \Delta;\ v[\cdot]] \quad (t_1 \leqslant t < t_2)$$

and we determine the guide's position at the instant $t = t_2$ from the condition

$$z_{*\Delta}[t_2] \in W_u^\vartheta(t_2) \cap G^{(u)}(z_{*\Delta}[t_1],\ v_*[\cdot])$$

assuming that this intersection is not empty.

If the condition $W_u^{\vartheta}(t_{i+1}) \cap G^{(u)}(z_{*\Delta}[t_i],\ v_*[\cdot]) \neq \phi$ is satisfied on the succeeding half-intervals $[t_i,\ t_{i+1})$ $(i = 2, 3, \ldots, N-1)$ as well, then the procedure indicated is carried out up to the instant $t = \vartheta$. If this condition is not satisfied, we can find an instant t_j at which first

$$W_u^{\vartheta}(t_j) \cap G^{(u)}(z_{*\Delta}[t_{j-1}],\ v_*[\cdot]) = \phi$$

Then from the condition $z_{*\Delta}[t_{j-1}] \in W_u^{\vartheta}(t_{j-1})$ and the definition of u-stability it follows that

$$M_{[t_{j-1},\, t_j]} \cap G^{(u)}(z_{*\Delta}[t_{j-1}],\ v_*[\cdot]) \neq \phi$$

i.e. an instant $t^* \in [t_{j-1},\ t_j]$ exists when the guide's position can be determined from the condition

$$z_{*\Delta}[t^*] \in M_{t^*} \cap G^{(u)}(z_{*\Delta}[t_{j-1}],\ v_*[\cdot])$$

Here

$$M_{[t_*,t^*]} = \{z : z = (t,\ \mu,\ \nu,\ x) \in M,\ \ t_* \leqslant t \leqslant t^*\}$$
$$M_{t^*} = \{z : z = (t^*,\ \mu,\ \nu,\ x) \in M\}$$

On the succeeding half-intervals $[t_i,\ t_{i+1})$ $(j \leqslant i \leqslant N-1)$ the controls $u[t]$ and $v_*[t]$ are determined by the relations

$$u[t] = u[s_\Delta^*[t_i];\ t_i,\ \Delta;\ u_*[\cdot]]$$
$$v_*[t] = v_*[s_\Delta^*[t_i];\ t_i,\ \Delta;\ v[\cdot]]$$

where $u_*[\cdot]$ is selected arbitrarily in $[t_i,\ t_{i+1})$ $(j \leqslant i \leqslant N-1)$.

The actual motion $(\mu_\Delta[t], \nu_\Delta[t], x_\Delta[t])$ and the guide's motion $(\mu_{*\Delta}[t], \nu_{*\Delta}[t], w_\Delta[t])$ which are realized on $[t_0,\ \vartheta]$ during the guide-control procedure are called approximation motions.

3. From the definition in Sect. 2 of the approximation procedure of control with a guide of the first player it follows that the perturbation $s_\Delta[t] = x_\Delta[t] - w_\Delta[t]$ on the half-interval $[t_i,\ t_{i+1})$ is described by the equation

$$ds_\Delta[t]\,/\,dt = As_\Delta[t] + B\,(p\,[s_\Delta^*[t_i],\ t_i] + R_0 s_\Delta^*[t_i] - u_*[t]) + C\,(v[t] - q\,[s_\Delta^*[t_i],\ t_i]) \tag{3.1}$$

where

$$p\,[s_\Delta^*[t_i],\ t_i] = p\,[s_\Delta^*[t_i];\ t_i,\ \Delta;\ u_*[\cdot]]$$
$$q\,[s_\Delta^*[t_i],\ t_i] = q\,[s_\Delta^*[t_i],\ t_i,\ \Delta;\ v[\cdot]]$$

The total derivative of the Liapunov function $\lambda(s)$ by virtue of Eq. (3.1) has the form

$$d\lambda(s_\Delta[t])\,/\,dt = (\partial\lambda\,/\,\partial s)'_{s=s_\Delta^*[t]}\,(As_\Delta[t] + B\,(p\,[s_\Delta^*[t_i],\ t_i] + R_0 s_\Delta^*[t_i] - u_*[t]) + C\,(v[t] - q\,[s_\Delta^*[t_i],\ t_i])) \tag{3.2}$$

on $[t_i,\ t_{i+1})$. From (3.2) there follows a relation valid for any two instants $t_{k_1}, t_{k_2} \in \Gamma_\Delta$

$$\int_{t_{k_1}}^{t_{k_2}} \left(d\lambda\,\frac{(s_\Delta[t])}{dt}\right) dt = \sum_{i=k_1}^{k_2-1} \int_{t_i}^{t_{i+1}} \left(d\lambda\,\frac{(s_\Delta[t])}{dt}\right) dt = \tag{3.3}$$

$$\sum_{i=k_1}^{k_2-1}\int_{t_i}^{t_{i+1}}\Big\{\Big(\partial\lambda\frac{(s)}{\partial s}\Big)_{s=s_\Delta^*[t_i]}B(p[s_\Delta^*[t_i],t_i]-u_*[t])+$$
$$\Big(\partial\lambda\frac{(s)}{\partial s}\Big)_{s=s_\Delta^*[t_i]}C(v[t]-q[s_\Delta^*[t_i],t_i])+$$
$$\Big(\partial\lambda\frac{(s)}{\partial s}\Big)_{s=s_\Delta^*[t_i]}A(s_\Delta[t]-s_\Delta^*[t_i])+\Big[\Big(\partial\lambda\frac{(s)}{\partial s}\Big)_{s=s_\Delta^*[t]}-$$
$$\Big(\partial\lambda\frac{(s)}{\partial s}\Big)_{s=s_\Delta^*[t_i]}\Big][As_\Delta[t]+B(p[s_\Delta^*[t_i],t_i]+R_0s_\Delta^*[t_i]-$$
$$u_*[t])+C(v[t]-q[s_\Delta^*[t_i],t_i])]+\beta(s_\Delta^*[t])\Big\}dt$$

To justify that the proposed guide-control procedure ensures the ε-proximity of the motions $w_\Delta[t]$ and $x_\Delta[t]$ we carry out the the upper bound of the integral in the left-hand side of the first equality in (3.3). It can be shown that the estimate

$$\int_{t_{k_1}}^{t_{k_2}}\Big(d\lambda\frac{(s_\Delta[t])}{dt}\Big)dt\leqslant\sum_{i=k_1}^{k_2-1}\{K(1+\|s_\Delta[t_i]\|)^2(\sqrt{\Delta}+\zeta)+ \qquad (3.4)$$
$$\beta(s_\Delta[t_i])\}\Delta+K(\sqrt{\Delta}+\zeta),\qquad K=\mathrm{const}>0$$

is valid. From this estimate, using stability theory arguments [2], we conclude: for any preselected $\varepsilon>0$ we can find numbers $\Delta(\varepsilon)>0$ and $\zeta(\varepsilon)>0$ such that every approximation motion $s_\Delta[t]$ with initial condition $\|s_\Delta[t_{k_1}]\|\leqslant\zeta(\varepsilon)$ will satisfy the inequality

$$\|s_\Delta[t]\|\leqslant\varepsilon \qquad (3.5)$$

for all $t\in[t_{k_1},t_{k_2}]$ provided that $\Delta\leqslant\Delta(\varepsilon)$ and $\|\Delta s_\Delta[t]\|\leqslant\zeta(\varepsilon)$.

Note that all the preceding arguments and the conclusion are valid only when the stabilizing part $r[s_\Delta^*[t]]$ of the first player's control $u[t]$ satisfies the constraint

$$I\equiv\int_{t_0}^{\vartheta}\|r[s_\Delta^*[t]]\|^2dt\leqslant\mu^2[t_0]-\mu_*^2[t_0]=\eta^2[t_0] \qquad (3.6)$$

From estimate (3.4) we see that for a finite time interval $[t_0,\vartheta]$ we can select the diameter Δ of partitioning Γ_Δ and the bound ζ on the phase vector measurement such that the control $r[s_\Delta^*[t]]=R_0s_\Delta^*[t]$ is admissible when $\|s_\Delta[t_{k_1}]\|\leqslant\zeta$, i.e. satisfies relation (3.6). In this case inequality (3.5) implies that the control procedure described ensures the ε-proximity of the motions $x_\Delta[t]$ of the actual system and $w_\Delta[t]$ of the guide, provided that the partitioning step Δ and the measurement error ζ are sufficiently small.

Note that in contrast to the estimate on the mismatch between the motions of the actual system and of the guide, given in [5], estimate (3.4) now does not contain an exponential factor. If, however, the system's phase vector is measured with an error, the presence of such an error leads, for a sufficiently large value of ϑ, to an excessive accumulation of the quantity I, i.e. to an accumulation violating inequality (3.6). However, if the first player's control is representable in the form $u=p+r$, where a part of the controlling force p is subject to an integral constraint and the other part r is subject to a geometric constraint, then a slight modification of the procedure presented above ensures the stable encounter of all approximation motions $x_\Delta [t]$ with the ε-neighborhood of set M^* by the instant ϑ denoting an arbitrarily large quantity.

The authors thank A. I. Subbotin for constant attention to this article and for valuable advice.

REFERENCES

1. Krasovskii, N. N., A differential game of encounter-evasion. I, II. Izv. Akad. Nauk SSSR, Tekhn. Kibernetika, Nos. 2 and 3, 1973.
2. Malkin, I. G., Theory of Stability of Motion. Moscow, "Nauka", 1966.
3. Krasovskii, N. N. and Subbotin, A. I., Position Differential Games. Moscow, "Nauka", 1974.
4. Reshetov, V. M., On a linear differential game of evasion. PMM Vol. 38, № 4, 1974.
5. Subbotin, A. I. and Ushakov, V. N., Alternative for an encounter-evasion differential game with integral constraints on the players' controls. PMM Vol. 39, № 3, 1975.

Translated by N. H. C.

UDC 62-50

SYNTHESIS OF TIME-OPTIMAL CONTROL OF A THIRD-ORDER OBJECT WITH A PHASE CONSTRAINT

PMM Vol. 40, № 3, 1976, pp. 446-454
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(Received June 23, 1975)

We examine a problem, arising in engineering practice, of the time-optimal control of a third-order linear object with a constraint on the control and on the phase coordinate . The synthesis of the control is described.

1. Statement of the problem. The following problem arises in the combined operation of two measuring devices tracking a moving object, each of which can track only in a certain part (action zone) of the space of measurements. From the information on the object obtained by the measuring device of the leaving zone, organize maximally quickly the tracking by the measurment device of the entering zone. For a number of measuring devices the dynamics of the tracking organization process can be described by a system of linear differential equations with constant coefficients and with constraints on the control u and on the phase coordinate (the action zone of the measuring device)

$$\frac{dx_1}{dt} = \Omega - x_3, \quad \frac{dx_2}{dt} = x_3, \quad \frac{dx_3}{dt} = -\frac{1}{T} x_3 + u \tag{1.1}$$

$$U \leqslant u < U, \quad x_{2_0}{}^2 - x_2{}^2 \leqslant 0 \tag{1.2}$$

The control $u^\circ(t)$, ensuring the satisfaction of condition (1.2) and translating the object (1.1) in a minimal time t_f from a specified initial point $x_1(0) = x_1^*$, $x_2(0) = x_2^*$, $x_3(0) = x^*_3$ (the initial position of the measuring device) to a specified final point $x_1(t_f) = 0$, $x_3(t_f) = \Omega$ (the condition for tracking to commence), is assumed